Heterogeneous Swarms with Reconfigurable Functions and Morphologies

Steven Ceron¹, Pascal Spino² and Daniela Rus¹

I. Introduction

Synchronization and swarming are two different types of self-organization present throughout many natural systems that have made their way into diverse artificial and robotic systems. On the one hand, synchronization refers to the self-organization of agents’ internal states (or phase) over time and has been found in diverse natural collective systems like flashing fireflies [1], chorusing frogs [2], and neuronal cell populations [3], [4]. In these synchronizing systems, each agent oscillates through some sort of cycle; this might mean flashing a light, making a sound, or sending an electrical pulse every few seconds. Each agent processes information about their neighbors’ internal states and adjusts the instantaneous rate at which it moves along its cycle so that it more closely matches the surrounding phases; the cumulative result of all constituents trying to match each others’ phases is that the collective synchronizes. Since the Kuramoto synchronization model was introduced in 1975 [5], researchers in essentially all fields have used this model and similar methods to study synchronization behavior [6] and engineers have used the same principles to design more robust power grid systems [7], better communication systems [8], and enable synchronization among groups of robots [9].

Swarming refers to the opposite; it is the self-organization of agents throughout space and is found throughout natural swarms at all length scales ranging from bacteria at the micron scale [10], to bridge-building ants at the centimeter scale [11], to flocking birds at larger scales [12]. Many models have been devised throughout the past few decades to study how natural, mobile, and distributed multi-agent systems may cluster, move together as a group, and disperse [13], [14]. Roboticists have used similar principles to create large groups of robots at the micron and macro length scales that can reconfigure into general and specific formations, work together to manipulate objects, and explore complex environments as mobile distributed sensor networks [15], [16], [17].

Although synchronization and swarming have independently enabled amazing developments in diverse research fields ranging from fundamental to application-level research, the effects of their mutual dependence remains largely unexplored. Igoshin et al. [18] and Iwasa et al. [19] took the first steps by introducing a set of models that demonstrated the emergent behaviors of a population of mobile agents that moved with respect to each other as a function of their internal phase and adjusted their internal state as a function of their motion with respect to each other. In 2017, O’Keeffe et al. [20] introduced a generalized Kuramoto model that officially began the swarmalator field (short for swarming coupled oscillators), and Ceron et al. [21] introduced a new swarmalator model whose emergent behaviors closely mimic the behavior of diverse systems like spermatozoa, social slime mold, and various microrobotic swarms.

Fig. 1: Introduction to swarmalators. (a) Phase values are between 0 and 2π and are mapped to a color bar. (b) Agents spatially attract toward other agents with a similar phase. (c) Agents inherently oscillate their phase at their natural frequency $\omega_i$ and couple to their neighbor’s phase through the term $K \sin(\theta_j - \theta_i)$. When $K > 0$, agents’ phase tend to move towards synchrony (top); when $K < 0$, agents’ phases tend to move towards asynchrony (bottom).

Studying the emergent behaviors of swarming coupled oscillators, or the mutual dependence between synchronization and swarming, holds enormous potential for researchers in a variety of fields ranging from biology to physics to engineering and has already begun with diverse studies related to the model’s emergent behaviors, the effect of various repulsive interactions, and the effect of confined trajectories for agents’ motions and interactions [22], [23]. Roboticists can use the same principles to design mechanisms for motion in a distributed and heterogeneous robot collective. Indeed, the first demonstration of the swarmalator model on robots showed that a group of 10 robots self-organized into formations like those theoretically predicted [24]. Moving forward, roboticists can consider the swarmalator model as an abstract mathematical framework that enables a group of heterogeneous robots to change the collective shape, motion, and inter-agent spacing without having to hardwire any behaviors into each of the robots. Each of the behaviors results from the dual position-phase pairwise interactions between coupling agents; this could enable a wide breadth...

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¹Computer Science and Artificial Intelligence Lab, Massachusetts Institute of Technology, Cambridge, MA 02139, USA. sceron@mit.edu, rus@csail.mit.edu
²Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA. spino@mit.edu
The model includes global attraction between all agents through a unit vector model and is tuned by the coefficient $A$, while distance-dependent repulsion is enabled by a power law model and tuned by the coefficient $B$. As shown in Fig. 1b, spatial-phase self-organization is driven by the coefficient $J \in [-1, 1]$, which enables attraction or repulsion between similar-phase agents. When $J > 0$, agents attract towards those with similar phases and when $J < 0$, agents attract towards those with a phase difference close to $\pi$. Eq. 2 is a variation of the Kuramoto model which defines each agent’s phase behavior as a function of the pairwise phase differences between agents, their relative positions, and a coupling coefficient $K$. As shown in Fig. 1c, each agent’s phase is mapped to a unit circle and it oscillates its phase at some inherent rate: its natural frequency $\omega_i$. $K$ enables agents to match each other’s phases (move towards synchrony) when it is positive, or move towards asynchrony when it is negative.

**A. Key Model Parameters**

Swarmalators’ motions are dependent on the pairwise phase interactions with their neighbors and their phase behavior is dependent on the relative motions with respect to each other. Fig. 2a-e details the five major behaviors introduced in the original swarmalator study [20] and by our new model (Fig. 2f-j) defined by Eqs. 1-2.

$$
\dot{x}_i = \frac{1}{N} \sum_{j \neq i}^N \left[ A \frac{x_j - x_i}{|x_j - x_i|} - \frac{x_j - x_i}{|x_j - x_i|^2} \left( B - J \cos(\theta_j - \theta_i) \right) \right]
$$

(1)

$$
\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j \neq i}^N \frac{\sin(\theta_j - \theta_i)}{|x_j - x_i|}
$$

(2)

Each agent $i$ follows an equation of motion $\dot{x}_i$ that is a function of its position $x_i$ with respect to its neighbor $j$ and its phase difference with its neighbor, where $\theta_{i,j} \in (0, 2\pi]$ and is representative of some internal state; each agent’s phase is represented by its color and is mapped to the colorbar in Fig. 1a.
III. EMERGENT BEHAVIORS AMONG BEACONS

When we consider many swarm robotics applications, the motion of agents with respect to each other is only half of the story: the agents are in some environment that they might react to and/or influence. If we take into account the environment and consider it as a network of immobile coupled oscillators (or beacons), we find that the mobile agents can be made to move in specific trajectories, construct bridges between specified points, and then deconstruct them; all of this is on-demand by changing parameter values in the swarmalator model so that specific beacons influence the heterogeneous collective’s behavior. This portion of our study is especially exciting since no other work in the coupled oscillators, swarming, or swarmalators field has considered the interactions between mobile and static coupled oscillators. Yet, many future robot swarms could benefit from this general formulation because it allows beacons to split the group by their heterogeneity and direct the various subgroups to specific locations through reactive phase interactions.

IV. AGGREGATION AND BRIDGE BUILDING

By setting the global parameters to \( K = 0 \) and \( J = 1 \) and the phases to a perfect uniform distribution between \( 0 \) and \( 2\pi \), we can enable the collective to form connections, or “bridges”, between specified points, as shown in Figs. 3a-c. In these experiments, the beacons are evenly distributed along the circumference of a circle with radius equal to 1 and their phases are organized incrementally in the counterclockwise position. This enables the collective to form a continuous bridge around the perimeter while changing its overall shape with the distribution of beacons. Note that when there are a low number of beacons (\( \sim 2 - 5 \)) the collective forms its bridge so that it almost intersects with the beacon’s location; however, at higher beacon numbers (100), the collective deviates away from the beacons’ locations because of their cumulative repulsion. When the collective forms a bridge between two points (Figs. 3a-b), the distribution of points and distinct number of bridges changes between 2 and 3. As shown in Figs. 3a-b, there is always a continuous bridge between the two beacons; this is because the left and right beacons always have an offset of \( \pi \) which means that the mobile agents will have to form at least two bridges between the two points since the mobile agents have a perfect distribution of phases between \( 0 \) and \( 2\pi \). When there are three bridges, there is always a continuous line of agents between the two beacons and a sparse distribution of agents around the perimeter. The sparsity of the agents along these regions is because only half of the collective is distributed across the second and third bridges and the system reaches an equilibrium point where approximately one quarter of the collective remains on one side of the connecting bridge and another quarter on the opposite side. It is important to note that the phase interactions are essentially attractive and repulsive potential fields between mobile agents and beacons; therefore, if there are just a few mobile agents on one side of the connecting line, which is the case in Fig. 3a, the attractive potential field from the bridge on the other side of the middle connecting line can slowly pull the mobile agents over.

As shown in Fig. 3a-f, the number of bridges and their overall organization can be controlled by managing the phase distributions. Throughout these experiments, we automatically generate the phase values of the beacons and the mobile agents so that a single bridge is formed between any number of points. By creating a perfect distribution of phases between \( 0 \) and \( \pi \), and arranging the phases of the beacons so they are increasing from left to right, the collective forms a set of bridges between all beacon locations. We demonstrate this behavior scales up with increasing numbers of beacons in Figs. 3d-f.

We demonstrate how this framework can be used to aggregate clusters at desired locations and form sparser bridges between beacon locations. This enables the agents to lose any global attraction (not related to phase interactions) between mobile agents and any repulsion (not related to phase interactions) between mobile agents and beacons. This enables the mobile agents to spread out around the beacon locations. In these cases the mobile agents and beacons each have a perfect distribution of phases between \( 0 \) and \( 2\pi \). Since \( K = 1 \) and \( J = 1 \), the collective splits into several
subgroups that synchronize within themselves and to each of the
designated beacons; however, if \( K \) were further raised, then eventually the whole collective would synchronize and
aggregate around one of the beacons. Therefore, using a single global parameter, we can enable a heterogeneous
collective to spatially split into its different subgroups or join
together into one large group. It should be noted that while it
would be beneficial to control the number of beacons across
which the collective distributes itself by simply changing \( K \)
to some value between 1 and the point where it globally
synchronizes, the transition from phase group synchronization
to global synchronization is a swift transition and does not allow
for this middle-case behavior where only part of the
collective synchronizes with some of the phases. There
are several ways to design bridges between certain beacons
or clusters at certain locations. One method is to modulate
the phase or natural frequency values of the beacons so that
part of the collective is attracted to those locations while
another part of the collective is attracted to another set of
locations, and the second is to automatically choose which
beacons are active and have phase interaction with the mobile
agents while controlling the distribution of phases among
the beacons; we choose the latter since varying the natural
frequency can increase the parameter space and can cause the
collective to have oscillatory motions between beacon
locations.

Throughout each of the experiments in Figs. 3, the natural
frequency for all mobile agents and beacons was kept at zero
to simplify the problem. Another variable that we controlled
throughout these experiments was the spatial organization of
phases with respect to the beacons; with the exception of the
randomly placed beacons, the beacons’ phases increased
from left to right (Figs. 3a-c and g-l). This enabled the collective to
create organized links between adjacent beacons in Figs. 3a-f
and i-j; however, if the beacons are unorganized, then
links between non-adjacent beacons can be formed by the
mobile agents. For example, if the phases of four beacons
are disorganized (not increasing in the counterclockwise
direction) then the collective is able to form an ‘x’ formation
by forming bridges between beacons on opposing sides.
To demonstrate a practical application of these behaviors,
we demonstrate a collective constructing and deconstructing
bridges along a set of beacons arranged in a circular con-
figuration with phases automatically generated as alternating
between 0 and \( \pi \) so the collective can easily form connec-
tions between neighboring beacons (Figs. 3k-l). By tuning
the global parameters of attraction and repulsion between the
agents, we can program the pairwise forces between mobile
agents which in turn affects their neighbor spacing. This
enables the collective to resemble a rod (Fig. 3k) or sparse
group (Fig. 3l) bending from one beacon to the next; this
model translates to three dimensions as well, which opens up
possibilities for its application to heterogeneous collectives
in which some agents move along a plane while other agents
move in 3D.

V. Conclusion

While this short paper advances the motion and phase
behavior control of reactive swarming coupled oscillators,
there are several areas for next steps and future work. First,
we must demonstrate the new model’s collective behaviors
on physical hardware. This requires building a large number
of robots that follow the swarmlator model to create the for-
mations and motions we report. We are currently developing
the physical platform that can implement these behaviors
for large groups (> 10) of robots since it is important
to consider the effects of scaling up in a physical robot
collective. Second, we show a set of experimental results
that are consistent across several trials. We plan to develop
formal analyses that prove the collective will always end
up at the same final formation or motion given any initial
configuration of phases and positions; we deem this to be
beyond the scope of this paper and plan to address this in
future work. Finally, we present an equation of motion that
does not consider any specific vehicle’s dynamics or velocity
and sensor limitations. This could be a weakness since
each robotic system may exhibit slightly different collective
behaviors given its constraints, but it could also be a strength
since it means the model is general and could be adapted
to diverse systems in which heterogeneous robot collectives
have different modes of locomotion. Our swarming coupled
oscillators are enabling a novel coordination mechanism
and currently, little is known about the collective behaviors
possible with swarmlators. We must begin with this general
model and explore what behaviors are possible before adding
in any constraints. Even with these limitations, this work
is a big step in the swarmlators field towards enabling
robot swarms that require very little information transfer
compared to the collective behavior complexity. Through this
work, we introduce the robotics community to swarmlators
so that swarm robotics researchers can expand upon this
model to enable more complex collective behaviors with
heterogeneous teams of robots.

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