Heterogeneous Swarms with Reconfigurable Functions and Morphologies

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I. INTRODUCTION

Synchronization and swarming are two different types of self-organization present throughout many natural systems that have made their way into diverse artificial and robotic systems. On the one hand, synchronization refers to the self-organization of agents' internal states (or phase) over time and has been found in diverse natural collective systems like flashing fireflies [1], chorusing frogs [2], and neuronal cell populations [3], [4]. In these synchronizing systems, each agent oscillates through some sort of cycle; this might mean flashing a light, making a sound, or sending an electrical pulse every few seconds. Each agent processes information about their neighbors' internal states and adjusts the instantaneous rate at which it moves along its cycle so that it more closely matches the surrounding phases; the cumulative result of all constituents trying to match each others' phases is that the collective synchronizes. Since the Kuramoto synchronization model was introduced in 1975 [5], researchers in essentially all fields have used this model and similar methods to study synchronization behavior [6] and engineers have used the same principles to design more robust power grid systems [7], better communication systems [8], and enable synchronization among groups of robots [9].

Swarming refers to the opposite; it is the self-organization of agents throughout space and is found throughout natural swarms at all length scales ranging from bacteria at the micron scale [10], to bridge-building ants at the centimeter scale [11], to flocking birds at larger scales [12]. Many models have been devised throughout the past few decades to study how natural, mobile, and distributed multi-agent systems may cluster, move together as a group, and disperse [13], [14]. Roboticists have used similar principles to create large groups of robots at the micron and macro length scales that can reconfigure into general and specific formations, work together to manipulate objects, and explore complex environments as mobile distributed sensor networks [15], [16], [17].

Although synchronization and swarming have independently enabled amazing developments in diverse research fields ranging from fundamental to application-level research, the effects of their mutual dependence remains largely unexplored. Igoshin et al. [18] and Iwasa et al. [19] took the first steps by introducing a set of models that demonstrated the emergent behaviors of a population of mobile agents that moved with respect to each other as a function of their internal phase and adjusted their internal state as a function of their motion with respect to each other. In 2017, O'Keeffe et al. [20] introduced a generalized Kuramoto model that officially began the swarmalator field (short for swarming coupled oscillators), and Ceron et al. [21] introduced a new swarmalator model whose emergent behaviors closely mimic the behavior of diverse systems like spermatozoa, social slime mold, and various microrobotic swarms.

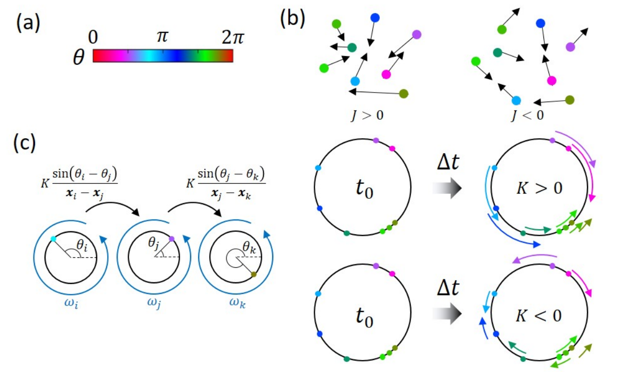

Fig. 1: Introduction to swarmalators. (a) Phase values are between 0 and 2π and are mapped to a color bar. (b) Agents spatially attract toward other agents with a similar phase. (c) Agents inherently oscillate their phase at their natural frequecny ω_i and couple to their neighbor's phase through the term $K\sin(\theta_j-\theta_i)$. When $K>0$, agents' phase tend to move towards synchrony (top); when $K<0$, agents' phases tend to move towards asynchrony (bottom).

Studying the emergent behaviors of swarming coupled oscillators, or the mutual dependence between synchronization and swarming, holds enormous potential for researchers in a variety of fields ranging from biology to physics to engineering and has already begun with diverse studies related to the model's emergent behaviors, the effect of various repulsive interactions, and the effect of confined trajectories for agents' motions and interactions [22], [23]. Roboticists can use the same principles to design mechanisms for motion in a distributed and heterogeneous robot collective. Indeed, the first demonstration of the swarmalator model on robots showed that a group of 10 robots self-organized into formations like those theoretically predicted [24]. Moving forward, roboticists can consider the swarmalator model as an abstract mathematical framework that enables a group of heterogeneous robots to change the collective shape, motion, and inter-agent spacing without having to hardwire any behaviors into each of the robots. Each of the behaviors results from the dual position-phase pairwise interactions between coupling agents; this could enable a wide breadth

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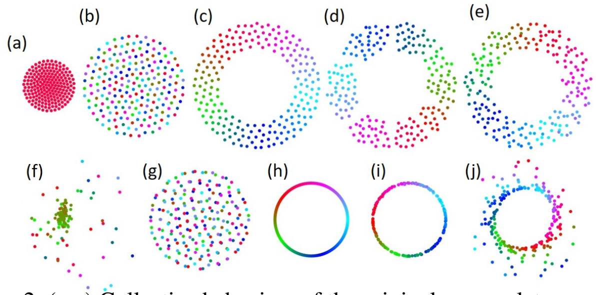

Fig. 2: (a-e) Collective behaviors of the original swarmalator model. (a) Synchronized cluster ($K = 1$, $J = 1$). (b) Asynchronous cluster ($K = 0$, $J = -1$). (c) Static phase wave ($K = 0$, $J = 1$). (d) Splintered phase wave ($K = -0.2$, $J = 1$). (e) Active phase wave ($K = -0.4$, $J = 1$). (f-j) Collective behaviors of the new swarmalator model. (f) Disorganized partially synchronized cluster ($K = 1$, $J = 1$). (g) Asynchronous cluster with paired agents ($K = 0$, $J = -1$). (h) Static phase wave ($K = 0$, $J = 1$). (i) Active phase wave ($K = -0.4$, $J = 1$). (j) Disordered Active phase wave ($K = -1$, $J = 1$).

of collective behaviors useful for robot swarms at the micron and macro length scales.

The full development of this work in simulation and in physical demonstrations will represent a major thrust in the swarming coupled oscillator field. Here, we introduce a new version of the swarmalator model that is more closely applicable to real-world robotic systems across a range of length scales, introduce the interaction between mobile and immobile coupled oscillators, and explore the automatic generation of an environment that enables a swarm to change its morphology, move along a specified trajectory, and form bridge-like structures between specified locations. Throughout this short paper we introduce the reader to the swarmalator field and its possible applications, study the emergent behaviors of our model in diverse settings, and experiment with user-specified behaviors that enable a heterogeneous collective to complete a specific task.

II. The Model

Swarmalators' motions are dependent on the pairwise phase interactions with their neighbors and their phase behavior is dependent on the relative motions with respect to each other. Fig. 2a-e details the five major behaviors introduced in the original swarmalator study [20] and by our new model (Fig. 2f-j) defined by Eqs. 1-2.

$$\dot{\mathbf{x}}_i = \frac{1}{N}\sum_{j\neq i}^{N}\left[A\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} - \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2}\left(B - J\cos\left(\theta_j - \theta_i\right)\right)\right] \tag{1}$$

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j\neq i}^{N}\frac{\sin\left(\theta_j - \theta_i\right)}{|\mathbf{x}_j - \mathbf{x}_i|} \tag{2}$$

Each agent i follows an equation of motion $\dot{\mathbf{x}}_i$ that is a function of its position $\mathbf{x}_i$ with respect to its neighbor j and its phase difference with its neighbor, where $\theta_{i,j} \in (0, 2\pi]$ and is representative of some internal state; each agent's phase is represented by its color and is mapped to the colorbar in Fig. 1a.

A. Key Model Parameters

The model includes global attraction between all agents through a unit vector model and is tuned by the coefficient A, while distance-dependent repulsion is enabled by a power law model and tuned by the coefficient B. As shown in Fig. 1b, spatial-phase self-organization is driven by the coefficient $J \in [-1, 1]$, which enables attraction or repulsion between similar-phase agents. When $J > 0$, agents attract towards those with similar phases and when $J < 0$, agents attract towards those with a phase difference close to π. Eq. 2 is a variation of the Kuramoto model which defines each agent's phase behavior as a function of the pairwise phase differences between agents, their relative positions, and a coupling coefficient K. As shown in Fig. 1c, each agent's phase is mapped to a unit circle and it oscillates its phase at some inherent rate: its natural frequency ω_i. K enables agents to match each other's phases (move towards synchrony) when it is positive, or move towards asynchrony when it is negative.

B. Model Behaviors

Our model is more realistic from a robotics perspective than previous models. As shown in Eq. 1, spatial phase interaction is controlled by the power law term. This enables agents to move towards or away from agents with similar phases as a function of their separation distance; this is important when imitating a real system or implementing on a distributed robotic system because agents will more reliably transmit information to other agents that are nearer to them.

As shown in Figs. 2f-j, our new model enables agents to have very small spacing between neighbors which causes them to form tight circles instead of annulus formations (Figs. 2h-i). Interestingly, one of the most disorganized-looking states is when $K = 1$, $J = 1$ (Fig. 2f); instead of forming a compact synchronized cluster, the collective has a group of agents that attract to each other and form an irregular shape while a number of agents remain on the outskirts of the collective. This disorganized behavior happens because the agents synchronize as a function of distance, and since they do not spatially attract to each other equally on the basis of their phase interactions, it is much more difficult for the agents to move toward other agents when they have a similar phase but are far away; this in turn reduces the agents' ability to work together to synchronize with other agents that have a large phase difference with them. When $K = 0$ and $J = -1$, the agents attract others with a large phase difference and form a phase-disorganized circular cluster similar to the one in Fig. 2j; however, agents with a large phase difference tend to pair with one another because of the high attraction that occurs when two agents are spatially close. Once two agents attract to each other, they increase the distance from other agents which decreases the effect of the spatial-phase interaction term in the equation of motion. This is the same mechanism that enables the collective to form tight circular formations in Figs. 2h-i. The agents are all attracted to each other through the unit vector portion of the equation of motion (this creates the circular

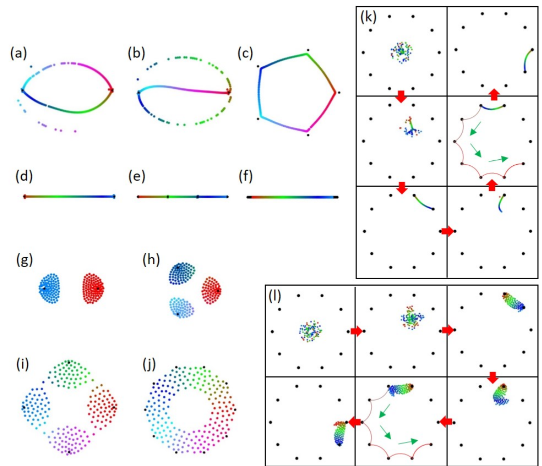

Fig. 3: Emergent behaviors of swarmalators in the presence of beacons. (a-c) Beacons organized by phase to increase the mobile agents' spatial phase order. Number of beacons: (a) 2; (b) 2; (c) 5. $K = 0$ and $J = 1$ for (a-c). (d-f) Bridges are formed between beacons when they are placed along a straight line. Number of beacons: (d) 2; (e) 4; (f) 100. (g-j) Mobile agents form static clusters around the beacons; $K = 1$ and $J = 1$. Number of beacons: (g) 2; (h) 3; (i) 4; (j) 10. (k-l) Collective constructs and deconstructs bridges between neighboring beacons arranged in a circular configuration; $K = 0$ and $J = 1$; sequential time steps shown correspond to $t = 1$, $t = 25$, $t = 100$, $t = 150$, $t = 200$, $t = 900$. (k) $A = 1$, $B = 1$. (l) $A = 2$, $B = 2$.

formation) but high attraction/high repulsion to other agents with a similar phase/large phase difference when they are near. The final state, the disordered active phase wave in Fig. 1m shows how the anti-phase coupling behavior of the swarmalators allows the agents to spread out more as they move around the perimeter of the formation. Note that the inner part of formation remains empty since the cumulative repulsion of agents with differing phases pushes the agents to move on the outer edge where there is more space.

III. Emergent Behaviors Among Beacons

When we consider many swarm robotics applications, the motion of agents with respect to each other is only half of the story; the agents are in some environment that they might react to and/or influence. If we take into account the environment and consider it as a network of immobile coupled oscillators (or beacons), we find that the mobile agents can be made to move in specific trajectories, construct bridges between specified points, and then deconstruct them; all of this is on-demand by changing parameter values in the swarmalator model so that specific beacons influence the heterogeneous collective's behavior. This portion of our study is especially exciting since no other work in the coupled oscillators, swarming, or swarmalators field has considered the interactions between mobile and static coupled oscillators. Yet, many future robot swarms could benefit from this general formulation because it allows beacons to split the group by their heterogeneity and direct the various subgroups to specific locations through reactive phase interactions.

IV. Aggregation and Bridge Building

By setting the global parameters to $K = 0$ and $J = 1$ and the phases to a perfect uniform distribution between 0 and 2π, we can enable the collective to form connections, or "bridges", between specified points, as shown in Figs. 3a-c. In these experiments, the beacons are evenly distributed along the circumference of a circle with radius equal to 1 and their phases are organized incrementally in the counterclockwise position. This enables the collective to form a continuous bridge around the perimeter while changing its overall shape with the distribution of beacons. Note that when there are a low number of beacons ($\sim 2-5$) the collective forms its bridge so that it almost intersects with the beacon's location; however, at higher beacon numbers (100), the collective deviates away from the beacons' locations because of their cumulative repulsion. When the collective forms a bridge between two points (Figs. 3a-b), the distribution of points and distinct number of bridges changes between 2 and 3. As shown in Figs. 3a-b, there is always a continuous bridge between the two beacons; this is because the left and right beacons always have an offset of π which means that the mobile agents will have to form at least two bridges between the two points since the mobile agents have a perfect distribution of phases between 0 and 2π. When there are three bridges, there is always a continuous line of agents between the two beacons and a sparse distribution of agents around the perimeter. The sparsity of the agents along these regions is because only half of the collective is distributed across the second and third bridges and the system reaches an equilibrium point where approximately one quarter of the collective remains on one side of the connecting bridge and another quarter on the opposite side. It is important to note that the phase interactions are essentially attractive and repulsive potential fields between mobile agents and beacons; therefore, if there are just a few mobile agents on one side of the connecting line, which is the case in Fig.3a, the attractive potential field from the bridge on the other side of the middle connecting line can slowly pull the mobile agents over.

As shown in Fig. 3a-f, the number of bridges and their overall organization can be controlled by managing the phase distributions. Throughout these experiments, we automatically generate the phase values of the beacons and the mobile agents so that a single bridge is formed between any number of points. By creating a perfect distribution of phases between 0 and π, and arranging the phases of the beacons so they are increasing from left to right, the collective forms a set of bridges between all beacon locations. We demonstrate this behavior scales up with increasing numbers of beacons in Figs. 3d-f.

We demonstrate how this framework can be used to aggregate clusters at desired locations and form sparser bridges between beacon locations. This enables the agents to lose any global attraction (not related to phase interactions) between mobile agents and any repulsion (not related to phase interactions) between mobile agents and beacons. This enables the mobile agents to spread out around the beacon locations. In these cases the mobile agents and beacons each have a perfect distribution of phases between 0 and 2π. Since $K = 1$ and $J = 1$, the collective splits into several subgroups that synchronize within themselves and to each of the designated beacons; however, if K were further raised, then eventually the whole collective would synchronize and aggregate around one of the beacons. Therefore, using a single global parameter, we can enable a heterogeneous collective to spatially split into its different subgroups or join together into one large group. It should be noted that while it would be beneficial to control the number of beacons across which the collective distributes itself by simply changing K to some value between 1 and the point where it globally synchronizes, the transition from phase group synchronization to global synchronization is a swift transition and does not allow for this middle-case behavior where only part of the collective synchronizes with some of the phases. There are several ways to design bridges between certain beacons or clusters at certain locations. One method is to modulate the phase or natural frequency values of the beacons so that part of the collective is attracted to those locations while another part of the collective is attracted to another set of locations, and the second is to automatically choose which beacons are active and have phase interaction with the mobile agents while controlling the distribution of phases among the beacons; we choose the latter since varying the natural frequency can increase the parameter space and can cause the collective to have oscillatory motions between beacon locations.

Throughout each of the experiments in Figs. 3, the natural frequency for all mobile agents and beacons was kept at zero to simplify the problem. Another variable that we controlled throughout these experiments was the spatial organization of phases with respect to the beacons; with the exception of the randomly placed beacons, the beacons' phases increased from left to right (Figs. 3d-f) or in the counterclockwise direction (Figs. 3a-c and g-l). This enabled the collective to create organized links between adjacent beacons in Figs. 3a-f and i-j; however, if the beacons are unorganized, then links between non-adjacent beacons can be formed by the mobile agents. For example, if the phases of four beacons are disorganized (not increasing in the counterclockwise direction) then the collective is able to form an 'x' formation by forming bridges between beacons on opposing sides. To demonstrate a practical application of these behaviors, we demonstrate a collective constructing and deconstructing bridges along a set of beacons arranged in a circular configuration with phases automatically generated as alternating between 0 and π so the collective can easily form connections between neighboring beacons (Figs. 3k-l). By tuning the global parameters of attraction and repulsion between the agents, we can program the pairwise forces between mobile agents which in turn affects their neighbor spacing. This enables the collective to resemble a rod (Fig. 3k) or sparse group (Fig. 3l) bending from one beacon to the next; this model translates to three dimensions as well, which opens up possibilities for its application to heterogeneous collectives in which some agents move along a plane while other agents move in 3D.

V. Conclusion

While this short paper advances the motion and phase behavior control of reactive swarming coupled oscillators, there are several areas for next steps and future work. First, we must demonstrate the new model's collective behaviors on physical hardware. This requires building a large number of robots that follow the swarmalator model to create the formations and motions we report. We are currently developing the physical platform that can implement these behaviors for large groups (> 10) of robots since it is important to consider the effects of scaling up in a physical robot collective. Second, we show a set of experimental results that are consistent across several trials. We plan to develop formal analyses that prove the collective will always end up at the same final formation or motion given any initial configuration of phases and positions; we deem this to be beyond the scope of this paper and plan to address this in future work. Finally, we present an equation of motion that does not consider any specific vehicle's dynamics or velocity and sensor limitations. This could be a weakness since each robotic system may exhibit slightly different collective behaviors given its constraints, but it could also be a strength since it means the model is general and could be adapted to diverse systems in which heterogeneous robot collectives have different modes of locomotion. Our swarming coupled oscillators are enabling a novel coordination mechanism and currently, little is known about the collective behaviors possible with swarmalators. We must begin with this general model and explore what behaviors are possible before adding in any constraints. Even with these limitations, this work is a big step in the swarmalators field towards enabling robot swarms that require very little information transfer compared to the collective behavior complexity. Through this work, we introduce the robotics community to swarmalators so that swarm robotics researchers can expand upon this model to enable more complex collective behaviors with heterogeneous teams of robots.

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